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INTERACTIVE AERODYNAMICS ANALYSIS AND DESIGN PROGRAMS FOR USE IN THE UNDERGRADUATE ENGINEERING CURRICULUM

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ABSTRACT

A computer program for the aerodynamic design and analysis of airfoils is described. This program is intended for use in an undergraduate aerospace engineering curriculum. It allows the students to interactively change the airfoil shape, thickness and camber as well as flight conditions, and allows the student to design airfoils subject to a specified pressure distribution over the surface. The mathematical and physical formulation of the analysis and design phases of the program are described, and several sample applications are presented.

INTRODUCTION

Aerospace engineering is a multi-disciplinary field. Aerodynamics, a major discipline within this field, deals with two fundamental issues: (i) what are the aerodynamic loads (i.e. lift, drag, side force, moments) created by a given body shape? (ii) what type of body shape will produce a desired pressure distribution? A particular class of bodies of interest to the aerodynamicist is the lifting surface. Analysis and design of lifting surfaces is important in many applications such as the design of aircraft, rotors, propellers, turbines, pumps and compressors. A practicing aerospace engineer is required to be proficient both in the design and analysis of such configurations.

In most undergraduate curricula, because of the limitation of time, and driven by a desire to teach the students the "fundamentals" of aerodynamics, emphasis is usually placed only on the synthesis aspects of aerodynamics, which addresses question (i) above. The aerodynamic design is usually addressed, and only briefly, in capstone design courses on aircraft design, at the senior level.

At Georgia Tech, as part of a program funded by the National Science Foundation, an effort has recently begun on vertical integration of aerodynamic design concepts into the curriculum. This effort calls for the creation of courseware (i.e. software and lecture notes integrated into the software) dealing with aerodynamic design. At the sophomore and junior levels, under this plan, the students will be exposed to 2-D subsonic airfoil design. During the final quarter (or semester) of the junior year, the student will be exposed to 3-D subsonic lifting surface design. More complex and difficult material such as transonic airfoil and wing design will be taught at the senior level, either as an elective course or as an independent research course where the student studies design concepts under the direction of a faculty advisor.

This paper addresses the first phase of this effort, namely courseware for 2-D subsonic airfoil design suitable for sophomore and junior level students. The mathematical and numerical formulation is first briefly described. Several sample applications of the design methodology are then presented.

OVERVIEW OF THE ANALYSIS /DESIGN

Inviscid Analysis:

The aerodynamic analysis used in the present analysis and design is based on the well-known "Panel" method. A brief overview of this method, based upon the formulation given in Ref. 1, is described here.

We first assume that the flow over the airfoil is steady, inviscid, two-dimensional and incompressible. The compressibility and viscous effects are accounted for later. This flow is uniquely described by a stream function $\psi(x,y)$ obey the following partial differential equation:

$$\nabla^2 \psi = 0 \tag{1}$$

At large distances from the airfoil, the stream function reduces to the sum of two components, one due to the freestream and the second due to the circulation over the airfoil Γ. The far field behavior is thus given by

$$\Psi(\bar{r}) = u_\infty y - v_\infty x - \frac{\Gamma}{2\pi}\log_e(|\bar{r}|) \tag{2}$$

Here, u_∞ and v_∞ are the Cartesian components of freestream velocity, and $\bar{r}$ is the position vector associated with the point (x,y).

The airfoil surface itself is a streamline as shown in Fig. 1, because the flow must be tangential to the surface. The value of the stream function at the solid surface, C, is an unknown and must be determined as part of the solution.

Because equation (1) is linear, its solution may be constructed using a superposition of several fundamental solutions. In the present work, we assume that the stream function is associated with a uniform freestream (which forms the first two terms on the right side of equation (2), and an infinitesimally thin vortex sheet γ surrounding the airfoil. This vortex sheet may be physically interpreted as the tangential velocity over the airfoil surface. The sheet strength has both a magnitude and a sign. The vortex sheet on the upper side of the airfoil is associated with clockwise vorticty, and is by definition, negative. The vortex sheet strength on the lower side is positive. By Bernoulli's equation, if the vortex sheet strength γ is known, the surface pressure coefficient may be computed as

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = 1 - \frac{\gamma^2}{V_\infty^2} \tag{3}$$

The superposed solution to the stream function is then given by

$$\psi(\bar{r}) = u_\infty y - v_\infty x - \frac{1}{2\pi}\oint_{Airfoil} \gamma(\bar{r}_o)\log_e\left[\left|\bar{r} - \bar{r}_o\right|\right]ds_0 \tag{4}$$

where the contour integral is over the vortex sheet surrounding the entire airfoil.

For computational purposes, we assume that the airfoil surface (and the vortex sheet) may be broken up into a total of N flat straight-line segments, called panels, and that the vortex sheet strength in each of such panels is constant, in a piece-wise sense. Then, the above integral becomes

$$\psi(\bar{r}) = u_\infty y - v_\infty x - \frac{1}{2\pi}\sum_{j=1}^{N}\gamma_j \int_j \log_e\left[\left|\bar{r} - \bar{r}_0\right|\right]ds_0 \tag{5}$$

These panels are usually numbered in a counterclockwise fashion. The first panel (j = 1) is located near the airfoil lower trailing edge, while the last panel (j = N) is located near the airfoil upper surface trailing edge, as shown in Fig. 2.

Now, we have a total of N+1 unknowns: the vortex sheet strengths γ of the N panels, and the value of stream function on the solid surface C.

These N+1 unknowns are solved by a collocation technique, where equation (5) is applied at a number of control points 'i' on the airfoil surface. For maximum accuracy, N control points are chosen, each located at the center of a panel. Equation () now reduces to

$$\psi(\bar{r}_i) = u_\infty y_i - v_\infty x_i - \frac{1}{2\pi}\sum_{j=1}^{N}\gamma_j \int_j \log_e\left[\left|\bar{r}_i - \bar{r}_0\right|\right]ds_o \tag{6}$$

We make note of the fact that $\psi(r_i)$ is C, and that the integral appearing in equation (6) may be calculated a priori. Equation (6) then becomes

$$u_\infty y_i - v_\infty x_i + \sum_{j=1}^{N} A_{i,j}\gamma_j = C$$

$$where,$$

$$A_{i,j} = -\frac{1}{2\pi}\int_j \log_e\left[\left|\bar{r}_i - \bar{r}_0\right|\right]ds_o \tag{7}$$

The quantity $A_{i,j}$ may be thought of as an influence coefficient, giving the stream function induced at control point 'i' by a vortex sheet of strength unity distributed over the panel 'j'. The integral defining $A_{i,j}$ may be analytically evaluated for flat panels. Equation (7) is a system of N linear equations for the N+1 unknowns. We need an extra equation to close the system. This extra equation is given by the Kutta condition, which states that the pressure on the upper and lower sides of the trailing edge be equal. Then, from equation (3)

$$\gamma_1 = -\gamma_N \tag{8}$$

Equation (7) and (8) provide N+1 equations for the N+1 unknowns. The matrix is system diagonally dominant and may be inverted using Gaussian elimination. After the vortex sheet strengths are computed, the surface pressures may be computed from equation (3). Finally, the airfoil lift c_l and pitching moment about the leading edge are computed by an integration of the surface pressures:

$$c_l = \int_{x=0}^{x=c}\left[c_{p,lower} - c_{p,upper}\right]d\left(\frac{x}{c}\right)$$

$$c_m = \int_{x=0}^{x=c}\left[c_{p,lower} - c_{p,upper}\right]\left(\frac{x}{c}\right)d\left(\frac{x}{c}\right) \tag{9}$$

where c is the airfoil chord length, and the origin is chosen to be at the airfoil leading edge.

Compressibility Effects:

The above analysis is strictly applicable for incompressible flows. The computed results may be corrected for compressibility effects at low subsonic Mach numbers M_∞ as follows:

$$c_p = \frac{c_{p,incompressible}}{\sqrt{1-M_\infty^2}}$$

$$c_l = \frac{c_{l,incompressible}}{\sqrt{1-M_\infty^2}}$$

$$c_m = \frac{c_{m,incompressible}}{\sqrt{1-M_\infty^2}} \qquad (10)$$

Viscous Effects:

In two-dimensional subsonic flows at low angles of attack, the drag is primarily due to skin friction, and may be computed using the von Karman momentum integral equation:

$$\frac{d\theta}{ds} + \frac{1}{u_e}\left(\frac{du_e}{ds}\right)[2+H]\theta = \frac{c_f}{2} \qquad (11)$$

In this equation, θ is the momentum thickness, u_e is the velocity at the edge of the boundary layer, and equals γ, H is the shape factor and c_f is the surface skin friction coefficient. This equation may be integrated along the airfoil surface, from the leading edge (s=0) to the airfoil trailing edge, using an explicit Runge Kutta scheme or an implicit scheme such as the Euler implicit scheme, provided appropriate empirically derived closure formulas for H and c_f are known. In the present work, a set of closure relations developed by Nash and McDonald [Ref. 2] were used in turbulent flow regions. In laminar flow regions, an alternate method known as Thwaites' method was used. Once the boundary layer characteristics are computed over the airfoil surface, the drag coefficient was computed using the Squire-Young formula [Ref. 3].

Aerodynamic Design:

As stated in the introduction, one of the objectives of the present effort is to enable the designer (the student) to arrive at an airfoil shape that generates a prescribed pressure distribution. Garabedian and McFadden recently [Ref.4] developed an iterative technique which changes the airfoil ordinate Y(x) by a small amount during each iteration. This change, ΔY is driven in their approach by the difference between the actual pressures over the airfoil and the target pressures. Malone, Vadyak and Sankar [Ref. 5] extended this approach to general configurations such as airfoils, wings and engine nacelles. This method has been applied by Malone et al to viscous airfoil design [Ref. 6], and extended by Silva and Sankar [Ref. 7] to the redesign of a wing for transonic cruise conditions. Recently this method has been recently generalized by Santos and Sankar [Ref. 8] to accept geometric constraints such as minimum and maximum limits on the airfoil thickness.

In this approach, the following ordinary differential equation is solved during each design update (or iteration):

$$A\frac{d^2(\Delta Y)}{dx^2} + B\frac{d(\Delta Y)}{dx} + C\Delta Y = (c_{p,actual} - c_{p,target}) \qquad (12)$$

The above ordinary equation is solved as a boundary value problem, replacing the ordinary derivatives using three-point finite difference approximations. One such equation per panel may be written. The resulting system of tri-diagonal equations may be easily inverted using a variation of the Gaussian elimination, known as the Thomas algorithm. The coefficients A, B and C are arbitrary coefficients that determine how rapidly the airfoil shape converges to the final design. When the computed Cp approaches the target values, the change in the airfoil shape ΔY between successive design updates goes to zero.

The above method works in compressible flows as well. If viscous effects are to be accounted for, the viscous displacement thickness $\delta^* = H\theta$ must be first computed from a boundary layer analysis, and subtracted from the designed effective airfoil shape.

RESULTS AND DISCUSSION

The airfoil analysis, boundary layer analysis and the design methods have been programmed for use in a junior level aerodynamics course. These analyses may be carried out on personal computers of the Macintosh IIci class. The CPU time required ranges from a few seconds (10 to 20) for an analysis with 40 panels over the airfoil, to a few minutes (2 to 3 minutes) for a design application.

The analysis code uses the standard Macintosh interface. Pull-down menus are used to specify the airfoil shapes to be used, as well as the flight conditions (angle of attack, Mach number). The airfoil shape which is the sum of a basic thickness distribution and a camber line shape may be interactively altered by the user. The program plots the airfoil shape as well as the computed pressure distributions. The student can thus quickly learn the cause and effects relationship between the airfoil shapes and the associated pressure distribution. Of course, the graphics images may be copied and pasted into other applications such as word processors and image generation/manipulation programs. Figure 3 showns a typical user interface on a Macintosh IIci system.

The boundary layer analysis can, in its present form, run only as a stand-alone application and has not been integrated into the analysis program. The user supplies the surface pressure distribution over an airfoil (from a previous panel analysis) by specifying the name of the data file where such information is stored. The program then prompts the user for flight conditions (Reynolds number, transition location). The program then prints out the computed displacement thickness, momentum thickness and shape factor over the upper and lower surfaces of the airfoil, along with the skin friction drag coefficient. The program must be repeatedly run once per angle of attack to generate the cl-cd drag polar associated with the airfoil.

The aerodynamic design program is also a non-interactive application. The user supplies a starting airfoil shape, and a target pressure distribution. The program then iteratively begins to change the airfoil shape. During each update of the airfoil shape, the intermediate shapes generated are written to a data file and saved, for later visualization of the intermediate airfoil shape. The design process stops when the airfoil produces a pressure distribution that is close to the target pressure distribution to within a user prescribed tolerance.

Figure 4 shows a NACA 4412 shape designed using the present methodology. A NACA 0012 airfoil was prescribed as a starting configuration, and the Cp distribution associated with the NACA 4412 airfoil was supplied as the target pressures. The design process reliably converged to the anticipated airfoil shape after 25 iterations .

Figure 5 shows a NACA 0012 airfoil designed using the present methodology, starting with a NACA 0006 airfoil. In this case, in about 40 iterations, the airfoil converged to the final (NACA 0012 shape).

At this writing, this program (written in Turbo-Pascal) is being ported to the Macintosh IIci system. We anticipate all the three programs to be available to the junior level students at Georgia Tech, beginning in the Fall of 1993.

CONCLUDING REMARKS

A family of airfoil analysis and design computer codes have been developed for undergraduate students, to expose them to the concepts of airfoil design. It is anticipated that these computer codes will be widely used by the students at the junior level because of their ease of use, short CPU time, and attractive user interface. We hope that these methodologies, packaged in this particular form, will allow the students them as numerical wind tunnels, and encourage them to design innovative airfoil shapes. Such airfoil shapes are of interest in a variety of applications such as propeller design, helicopter rotor design, high lift airfoil design, and natural laminar flow airfoil design.

ACKNOWLEDGMENTS

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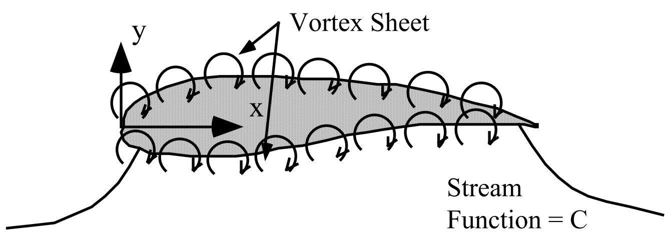
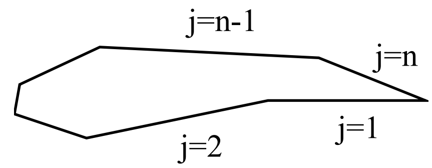


Figure 1. Overview of the airfoil and Vortex Sheet.



2. Panel Numbering System

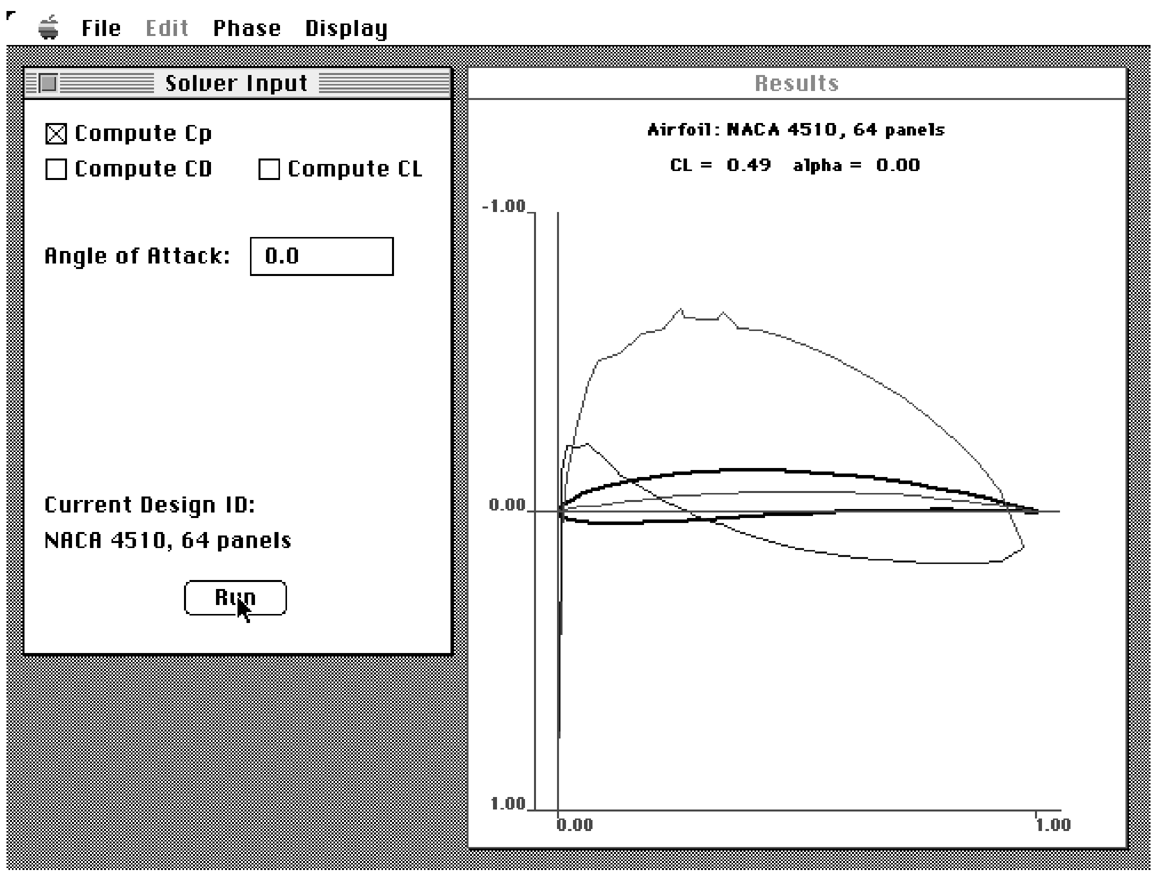


Figure 3. Typical screen display with results

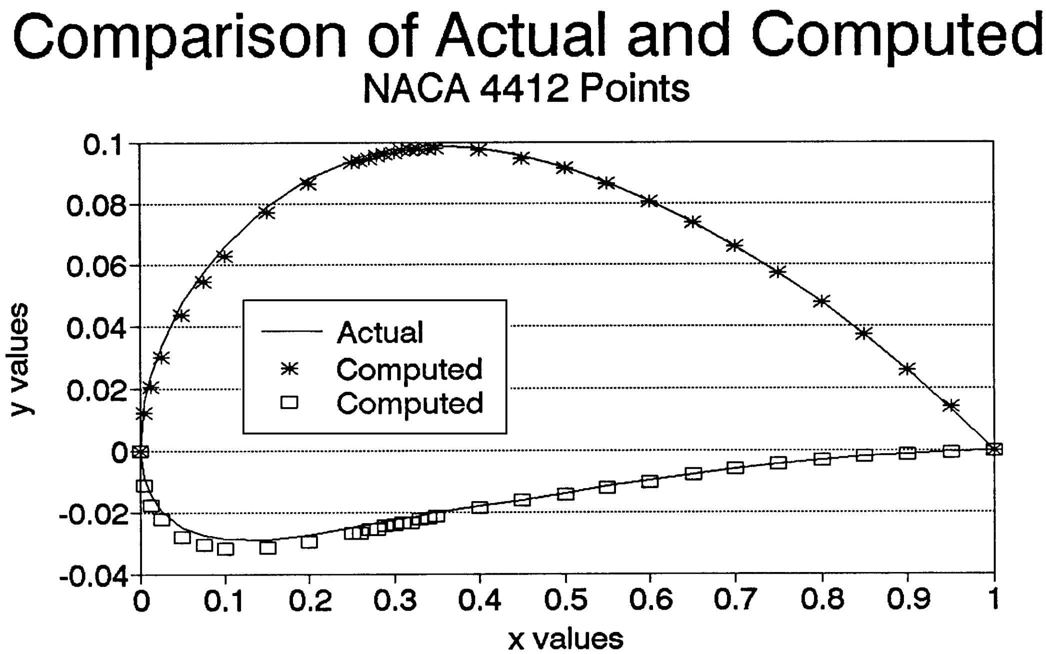


Fig. 4 Design of NACA Airfoil 4412.

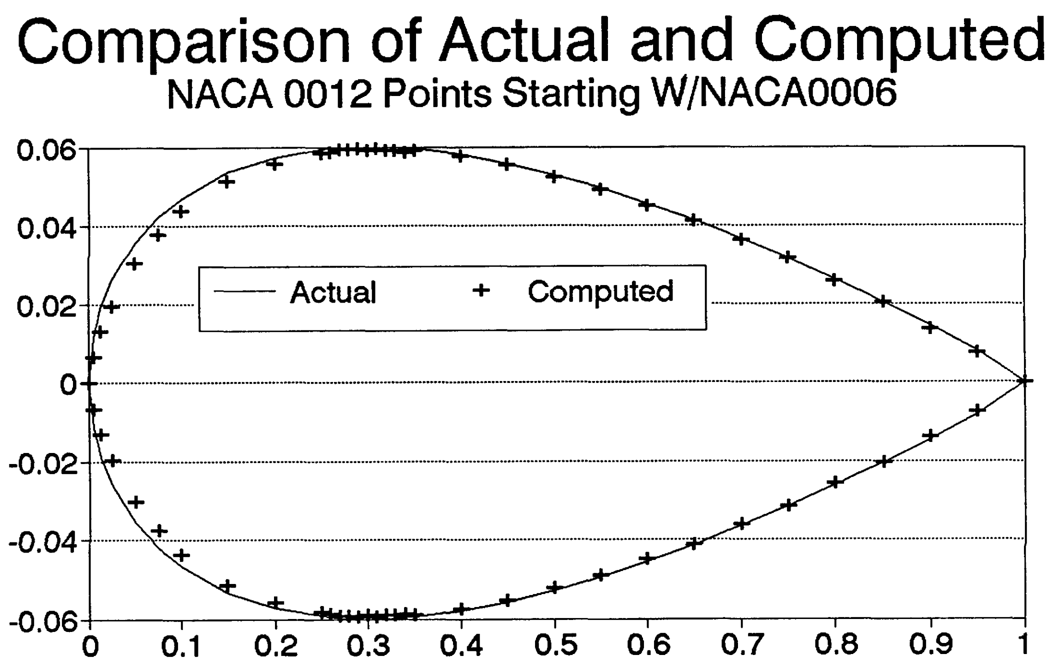


Fig. 5 Design of NACA Airfoil 0012.