

Vibration Visualization using Longitudinal Vibration Simulator (LVS)

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Abstract

Visualization of equations in space and time is crucial to understanding of vibrations. LVS (Longitudinal Vibration Simulator) is an analysis and educational tool for longitudinal vibrations. The LVS software gives visual feedback of the stress/strain field for free and forced vibration, with and without damping. The software allows the user to vary all pertinent parameters of the analysis and view the results in a color animation. The time history can be recorded so the stress/strain field for the time series under study can be viewed with a single graph. By using this visual feedback, LVS can increase and enhance understanding of various vibration phenomenon that are difficult to conceptualize.

Introduction

Recently there has been an increase in research in the area of computer integration into higher education. The results show increases in understanding, speed of learning, and interest in studying. Marcoulides has shown significant increases in student performance for a computer aided instruction program over traditional methods. In simulation software for statics and dynamics courseware, Chapra and Canale¹ have demonstrated the importance of visual images in the education process. Borkowski et al.² have shown the use of multimedia in the classroom allows the introduction of more material. Cobourn and Lindauer³ have developed a multimedia module for introductory thermodynamics that students claim aid the learning process. This research has shown that computer based education can have beneficial results when compared to traditional methods. Although not all students are inclined to learn through visualization, combining traditional methods with computer visualization will help homogenize the level of understanding in a class.

In the area of vibrations, results are given in space and time. A tool to visualize these results would be beneficial to the education process. Development of such a tool is the focus of the LVS project.

Design Goals

Courses in continuous systems vibration are generally taught in a highly theoretical form due to the nature of the material. In depth analysis of a system other than undamped, free vibration quickly becomes mathematically intense. However, the student should still have an understanding of the response of more complex and realistic systems. A computer program that performs the calculations

and returns the results can give the student insight into problems that are rarely even discussed except in very specialized courses. Linking complex theory and equations to numerical and animated graphical results increases understanding and information retention. LVS was developed with the idea that a simple to use program that could relate the complex equations involved to a physical result would be very beneficial.

Simplicity of use is directly related to the number of options available to the user. For this reason, LVS was developed with the single problem of longitudinal vibration in mind. All pertinent parameters such as beam length, material properties, geometry, etc. are set by clicking the appropriate buttons. The interface is a single screen that is clear and self explanatory.

From the standpoint of an Engineer, stress and strain are the most important results from an analysis. However, in vibration analysis these results are directly related to the normal modes of a structure, their natural frequencies, and how they interact together. The results obtained through LVS relate these visually as opposed to the mathematical derivation common in education. The mathematical derivation is still necessary but now there is a tool to gain insight into the physical meaning of these equations.

Operation of LVS

Performing an analysis with the LVS software is broken into three steps; beam construction, testing, and interpretation of results. Figure 1 shows the general layout of the program.

Beam Construction

The beam geometry, i.e. fixed-free, can be changed by selecting the desired configuration. The beam properties of length, Young's Modulus and density can be changed simply by selecting the current value and entering a new number.

Testing

Testing free or forced vibration response can be performed by selecting the desired type in the testing window. For forced vibration, the forcing function can be set by scrolling through the available types and changing the pertinent parameters such as peak force and time characteristics. Viscous damping can be turned on or off by selecting damped or undamped response. Each mode from 1 to 6 can then be turned on or off for the purpose of analysis. For most structures, only the first few modes are significant since the magnitude generally decreases with mode number. For free response, the mode participation factor can be

changed by the user by simply entering a new value. For damped response, the viscous damping coefficient for each mode can also be changed. Typical values for mode participation factors and damping coefficients are of the order of 0.01.

The testing time is set by entering the start time and end time in the corresponding locations. In order to simplify analyzing a single cycle, the times may be entered as a fraction. For example if the frequency of a mode is 26 Hz then a start time of 0.0 and an end time of 1/26 would result in the first full cycle of that mode. The number of steps determines where each calculation takes place by dividing the total testing time into equal time steps. A testing time of 1 sec. with 4 steps will give results at 0.0, 0.25, 0.5 and 0.75 sec.

Recording the time history will show the stress field for the entire testing time as a graph with the beam axis horizontal and testing time going from top to bottom. This gives a single picture view of the stress field and how it changes through time.

A displacement scale factor may be entered to increase the visibility of displacements. A value of 1 will make the displacements in scale with the beam. However, since most displacements are quite small, values with a couple of magnitudes of order should be used to observe the deflections at the end of the beam.

The maximum and minimum stress levels for the color plot can be changed like any other parameter. If a value of 30 MPa is entered, red and green will correspond to levels over 30 MPa in tension and compression respectively.

At any time, the user can use the Calc button to displace the natural frequency and period of the first 6 modes. This can be very useful when setting the testing time as discussed above.

Results

Once the calculations have been performed for each time step, the results are displayed as a animated stress/strain field. Controls much like those on a VCR are used to stop, play, or advance the animation by single frames. For each frame or time step, the maximum and minimum stress and strain are shown by a pointer and their values are displayed. The motion of these pointers during the animation give insight into how the field is changing with time.

Example Application

A bar with a sinusoidal end load is a classic example from vibration coursework. Many types of rotating machinery create forces of the sinusoidal type, the columns supporting the floor of such a machine would thus be subjected to a time varying load as shown in Fig. 2.

Designing such a column is an iterative process that requires knowing the maximum and minimum stress the beam

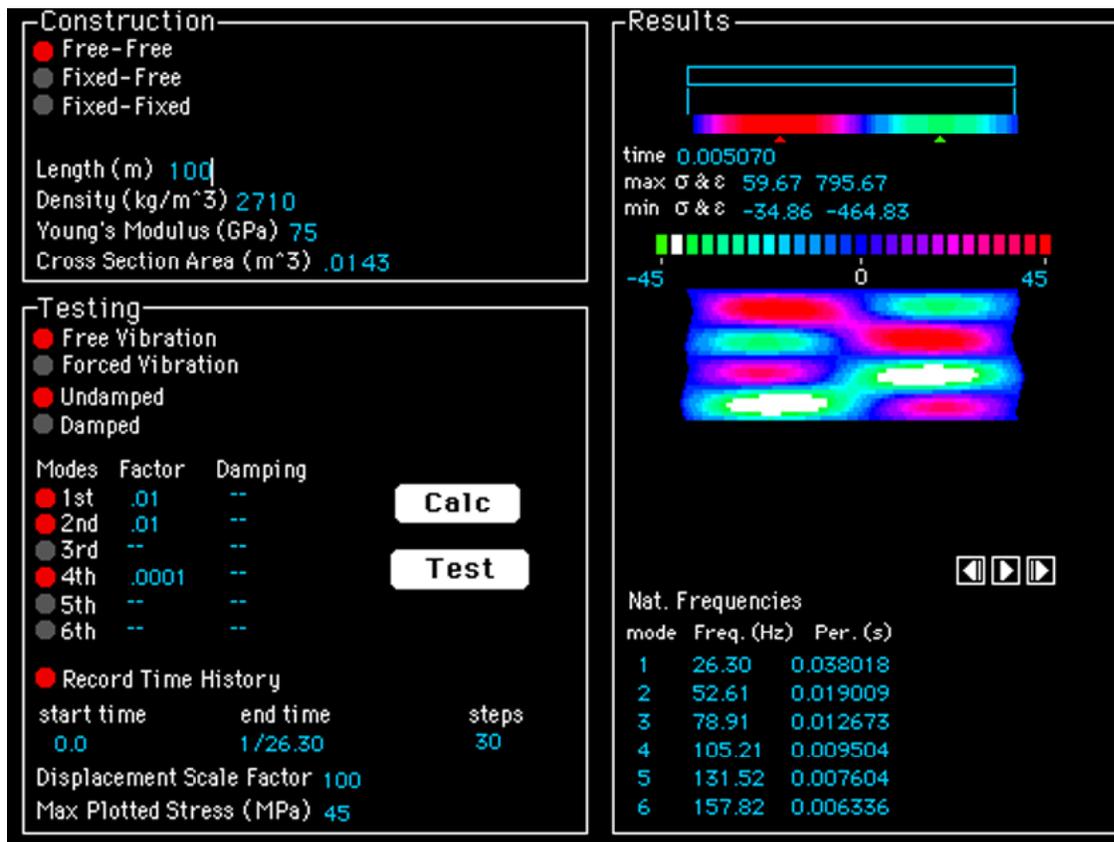


Figure 1. Screen Layout for LVS

will be subjected to. Also, fatigue analysis is directly related to the frequency and stress levels for the different modes of vibration. Using the LVS software simplifies the analysis of such a problem which is quite complex and solving by hand can be lengthy.

To start the analysis, first select fixed-free since it represents the sample case, then set the parameters for the column. Since there is a forcing function, select forced response with a sinusoidal load and set the maximum value to 100 and the frequency to 30. Looking at forced response generally requires the addition of damping to remove any transient effects so select damped response. For completeness, use all six modes and a typical damping ratio such as 0.005 for each mode. Select time history to get a single picture of the response. The steady state response is of primary interest so calculate the natural frequencies using the Calc button and enter the start time as the start of the tenth cycle and the end time as the start of the eleventh. For example, if the first natural frequency is 30 Hz, the start time will be 10/30 and the end time will be 11/30. For a smooth animation use 30 frames. So the displacements are visible, use a displacement scale factor of 100 and set the maximum stress to 50 MPa.

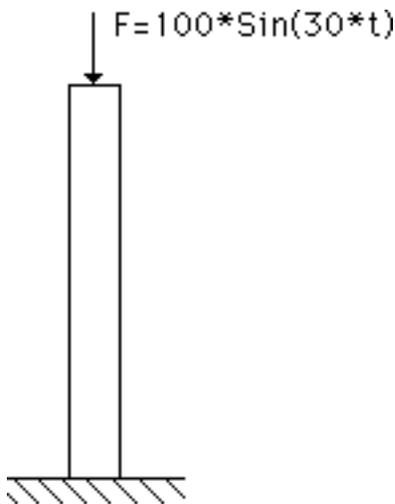


Figure 2. Sample Application

After all the testing parameters are set, pressing the Test button will start the calculations and each frame of the animation will be computed. Once the computations are finished, the animation will begin and stop/play controls can be used to view the results. The maximum and minimum stress values and their locations can now be easily pick off for each time step. Such analysis would be quite difficult and lengthy to perform by hand.

Development of LVS

LVS was developed using Macromind Director, a multimedia authoring tool. The mathematical speed of

Director is not ideally suited to engineering applications however visual interactivity was the primary concern for LVS. Extensive programming, called Lingo in Director, was developed to handle the computations required for vibration analysis and transfer the results to a graphical form for the user. Mathematica was used to find closed form solutions for some forced response systems, and also to check the accuracy of results obtained in Director.

The current version of Director allows linking to programs like Mathematica so parameters entered in the multimedia shell can be transferred to Mathematica for solution. However this requires the user to have a copy of Mathematica, a fairly expensive program. Testing has also shown that using this link has resulted in inconsistent results, i.e. crashing Mathematica or Director, or completely locking up the system. New versions will undoubtedly be more stable and thus more applicable. Ideally, the parameters could be entered and transferred to a program such as Mathematica which not only performs the calculation but also performs the graphing and returns the results.

Also, programs like Mathematica could be used to perform calculations for forcing functions that do not have a simple closed form solution. Mathematica could perform the necessary integrals instead of the solution being coded into the program itself. Currently, no such program exists with the mathematical tools and graphical interaction necessary to make such serious analysis a reality.

LVS currently runs on the Macintosh operating system and has been testing on machines ranging from a Quadra 700 to a PowerPC 7100 with as little as 4 MB. A Windows version of the program is planned for early in 1995. A free copy of the program is available from the authors.

Longitudinal Vibration Theory

This section describes the theoretical basis on which the LVS software is based. This is simplified look at free and forced response for longitudinal vibration and several steps and details have been omitted. For a complete analysis see Rao⁴.

Differential Equation, Boundary Conditions and Initial Conditions

For longitudinal vibration, the equation of motion is given by the partial differential equation

$$c^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = f(x, t)$$

The boundary conditions for fixed and free ends are given by

$$\text{free} \quad \frac{\partial u}{\partial x} = 0$$

$$\text{fixed} \quad u = 0$$

The initial conditions describe the original shape and motion of the beam. LVS assumes the initial conditions to be

$$u(x, 0) = 0$$

$$\frac{\partial u}{\partial t}(x, 0) = 0$$

Mode Summation

The longitudinal motion of a vibrating bar can be expressed as a infinite series sum of the normal modes.

$$u(x,t) = \sum_{n=1}^{\infty} C_n U_n(x) T_n(t)$$

In practice, the series is truncated at some number n. The LVS software has the capacity for n up to 6. Generally the first few modes are the only ones of interest and the series is limited to 6 for the speed and ease of computation. This truncated series gives the displacement of each point in space and time from which the strain and stress can be obtained.

The mode shapes (U_n 's) and natural frequencies (ω_n 's) depend on the beam geometry and are given by the following equations

Free – Free	$U_n(x) = \cos \frac{\omega_n x}{c}$	$\omega_n = \frac{n\pi c}{L}$
Fixed – Free	$U_n(x) = \sin \frac{\omega_n x}{c}$	$\omega_n = \frac{n\pi c}{L}$
Fixed – Fixed	$U_n(x) = \sin \frac{\omega_n x}{c}$	$\omega_n = \frac{(2n-1)\pi c}{2L}$

The time dependent terms of the series (T_n 's) depend on the initial conditions, the forcing function and whether the system has damping.

Free Vibration

Under free-vibration and zero initial conditions, T_n can be found to be

Undamped	$T_n(t) = \sin \omega_n t$
Damped	$T_n(t) = e^{-\zeta_n \omega_n t} \sin \omega_{dn} t$

where ζ is the viscous damping coefficient and ω_{dn} is the damped natural frequency given by

$$\omega_{dn} = \omega_n \sqrt{1 - \zeta_n^2}$$

Since there is no forcing function for free-vibration, the constants C_n cannot be determined and must be supplied.

Forced Vibration

For forced response, the C_n s are determined from the forcing function and are contained within each T_n so the response is given by

$$u(x,t) = \sum_0^{\infty} U_n(x) T_n(t)$$

where each U_n is defined as before.

For the forcing function $F(x,t)$, each T_n is found by solving the differential equation

$$\ddot{Y}_n + \omega_n^2 T_n = \frac{1}{M_n} \int_0^L F(x,t) U_n(x) dx$$

If we consider $F(x,t)$ to be separable, that is

$$F(x,t) = \frac{P_0}{L} p(x) f(t)$$

then each T_n , for zero initial conditions, is defined by the equation

$$\text{Undamped } T_n(t) = \left(\frac{P_0 \Gamma_n}{M_n \omega_n} \right) \int_0^t f(\xi) \sin \omega_n (t - \xi) d\xi$$

Damped
where

$$M_n = \int_0^L U_n^2(x) m(x) dx$$

$$\Gamma_n = \frac{1}{L} \int_0^L p(x) U_n(x) dx$$

and $m(x)$ is the mass distribution and $p(x)$ is the spatial part of the forcing function $F(x,t)$.

Implementation into Graduate and Undergraduate Courses

LVS is well suited as both a teaching aid and an analysis tool. Using LVS within a traditional lecture brings a visual element to a normally static result. The class can perform several what-if scenarios and discuss possible results then check their reasoning. Likewise, LVS can serve strictly as an analysis tool to perform calculations for some assignment, possibly in a design project. Ideally LVS would be implemented in the classroom as well as outside to receive full benefit of its visual representation and numerical ability.

Summary

Traditional teaching methods for vibrations employ static equations in which the results can sometimes be as unclear as the original problem. To alleviate this problem, a dynamic way of representing these results was developed and implemented in a software package called Longitudinal Vibration Simulator (LVS). By concentrating on a single problem, LVS is capable of operating with a simple user interface that shows all parameters and returns dynamic visual results that can be easily related to traditional mathematical derivations in order to increase understanding.

Marcoulides, G., Improving Learner Performance with Computer Based Programs, *J. Educational Computing Research*, 6, 1990, pp. 147-155.

¹Chapra, S.C. and Canale, R.P., Breaking Through Conceptual Roadblocks, *Academic Computing*, 1989, pp. 12-15, 45-51.

²Borkowski, C.A., Kulp, P.T., Luetzelschwab, M., Gile, M. and DiCesare, F., Role of Hypermedia Interactive environment in a laboratory Course, *Proceedings of the 22nd Annual Frontiers in Education Conference*, 1992, pp. 156-161.

³Cobourn, W.G. and Lindauer, G.C., A Flexible Multimedia Instructional Module for Introductory Thermodynamics, *Journal of Engineering Education*, July 1994, pp. 271-277.

⁴Rao, S.S., *Mechanical Vibrations*, 2nd edition, Addison-Wesley, 1990.